DRP-1-13
April 1994

Dredging Research Technical Notes

Tidal Constituent Database — East Coast, Gulf of Mexico, and Caribbean Sea

Purpose

This technical note describes a database of tidal elevation boundary conditions generated in support of "Long-Term Fate of Dredged Material Disposed in Open Water" research of the Dredging Research Program (DRP), being conducted by the U.S. Army Engineer Waterways Experiment Station (WES). The database described allows the user to manually generate time series of tidal elevations or to use a program to access the full database to generate time series of both tide elevations and currents for any location along the U.S. east coast, Gulf of Mexico, and Caribbean Sea. Although the capability to generate these time series was developed to provide input to the long-term fate and stability model LTFATE, the generated time series can be used for any application requiring tidal forcing data.

Background

The long-term fate research has been concerned with developing techniques to predict the long-term fate of dredged material after it has been deposited in open water on the ocean floor, that is, to address the question whether a dredged material disposal site, either existing or proposed, is dispersive or nondispersive (Scheffner 1992). If the site is dispersive, an additional capability of the model is to estimate the rate of erosion and fate of the material. Because sediment is primarily eroded and transported as a function of waves and currents, the approach is to construct databases of site-specific information that can be used as input to coupled hydrodynamic, sediment transport, and bathymetry change models for predicting the long-term behavior of disposal sites. In the DRP, attention has been focused on the development of the wave, tidal, and storm surge components.

The wave component of the database provides the capability for generating time series of wave height, period, and direction for any location at which a WES Wave Information Study hindcast is available. The wave simulation capability is described in Borgman and Scheffner (1991). A database of regional tropical and extratropical storm surge elevation (and current hydrographs and stage-frequency relationships) is currently under development. This technical note describes the tidal component.

Additional Information

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The Tidal Database

The tidal contribution to the overall database has been completed for the entire east coast of the United States, Gulf of Mexico, and Caribbean Sea (Westerink, Luettich, and Scheffner 1993). The tidal component is both co-tidal chart-based for manual simulation of time series and PC-based. Both versions are described briefly in this technical note.

This tidal database consists of precomputed amplitudes and Greenwich epochs corresponding to the eight primary diurnal and semidiurnal astronomical constituents (K_1, O_1, P_1, Q_1, M_2, S_2, N_2, and K_2). The constituents are based on a 6-month simulated tidal time series computed by the long-wave hydrodynamic finite element model ADCIRC (Luettich, Westerink, and Scheffner 1992). This model was developed for the DRP for the specific application to long-term computations over very large computational domains.

The tidal constituents were computed by an eight-constituent harmonic analysis of model elevation and current time series at each node of the computational grid shown in Figure 1. Boundary conditions for the simulation were provided at the mid-Atlantic Ocean boundary according to Schwiderski's (1980) global ocean database. Use of the DRP tidal database will allow the user to generate time series of tidal elevations and currents at any location within the computational domain for any time period—past, present, or future. Although the intent of the simulation capability is to provide time series input to the long-term fate model LTFATE, the generated data can be used for any application requiring tidal forcing (tidal circulation studies).

The following section describes the procedure used to generate time series of tidal data based on astronomical arguments, that is, amplitudes and epochs. Then, descriptions are given of applying the procedure with the DRP-generated database to either manually compute equilibrium tidal elevation time series or to use a computer-based program to generate simulated forecast or hindcast time series of surface elevations and currents.

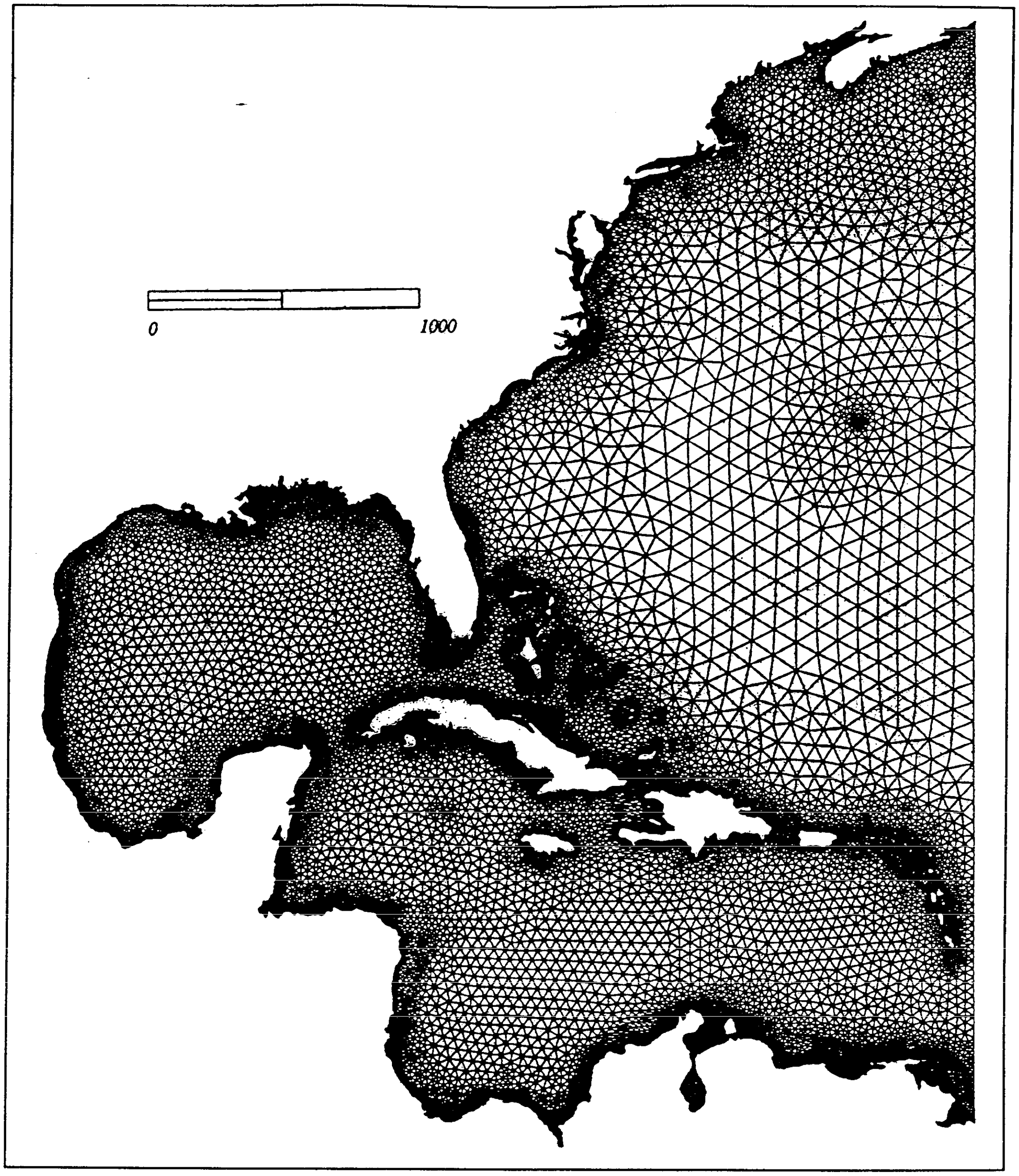

Figure 1. Tidal database computational domain

Harmonic Reconstruction of Tides

The tidal elevation and current time series at any location can be written as a function of known harmonic constituents according to the following general relationship:

$$h = H_0 + \sum_{n=1}^{N} f_n H_n \cos\left[a_n t + \left(V_o + u\right)_n - \kappa_n\right] \quad (1)$$

where

h = height of the tide at any time t

H_0 = mean water level above some defined datum such as mean sea level

N = total number of tidal constituents in the series reconstruction

f_n = node factor for reducing mean amplitude

H_n = mean amplitude of tidal constituent n

a_n = speed of constituent n in degrees/unit time

t = time measured from some initial epoch

$(V_0 + u)$ = value of the equilibrium argument for constituent n at some location when $t = 0$

κ_n = local epoch of constituent n

In the above formula, the tide is represented as the sum of a coefficient multiplied by the cosine of its respective arguments. A finite number of constituents are used in the reconstruction of a tidal signal. The values for the arguments are generally computed through least squares analyses of prototype data (Dronkers 1964). The National Oceanic and Atmospheric Administration's National Ocean Survey (NOS) generally uses 37 constituents in its published harmonic analyses. These results are usually based on an analysis of a minimum of 1 year of prototype data. As an example, a listing of the NOS constituents and the corresponding amplitudes and local epochs for the tide station located at Duck, NC, is shown in Table 1. A thorough description of each constituent is provided in Dronkers (1964) and Schureman (1958).

Most constituents in Table 1 are associated with a subscript that indicates the approximate number of cycles per solar day (24 hr). The constituents with subscripts of 2 are classified as semidiurnal constituents and produce a tidal contribution that occurs approximately twice a day. Diurnal constituents occur approximately once a day and have a subscript of 1. Symbols with no subscript are termed long-period constituents and have periods greater than a day.

The majority of constituents shown in Table 1 can be neglected along the east coast, Gulf of Mexico, and Caribbean Sea. For example, in the modeled domain of Figure 1, over 90 percent of the tidal energy can be represented by the M_2, S_2, N_2, and K_2 semidiurnal and K_1, O_1, P_1, and Q_1 diurnal constituents. For example, these eight constituents account for 94 percent of the energy of a signal computed for Duck, NC, with the full 37 NOS constituents. For this reason, the above eight constituents are used to define the tidal contribution to the east coast, Gulf of Mexico, and Caribbean Sea database. In other locations in the world, many more tidal constituents may be needed to adequately represent the tide. For

Table 1. NOS Tidal Constituents for Duck, North Carolina					
Symbol	**Amplitude H, ft**	**Epoch κ, deg**	**Symbol**	**Amplitude H, ft**	**Epoch κ, deg**
M_2	1.5520	207.31	Mm	0.0810	243.46
S_2	0.2990	231.23	Ssa	0.3120	27.93
N_2	0.3830	187.61	Sa	0.2340	205.48
K_1	0.2820	97.24	Msf	0.0400	81.43
M_4	0.0110	298.72	Mf	0.580	150.06
O_1	0.1910	117.75	ρ_1	0.0050	74.06
M_6	0.0320	139.39	Q_1	0.480	102.55
$(MK)_3$	0.0030	49.72	T_2	0.250	202.51
S_4	0.0110	301.48	R_2	0.0070	76.24
$(MN)_4$	0.0110	257.74	$(2Q)_1$	0.0060	113.27
ν_2	0.0640	188.44	P_1	0.1020	99.00
S_6	0.0060	308.41	$(2SM)_2$	0.0070	237.52
μ_2	0.0570	188.44	M_3	0.0130	167.08
$(2N)_2$	0.0600	174.42	L_2	0.0520	230.53
$(OO)_1$	0.0170	117.41	$(2MK)_3$	0.0060	125.43
λ_2	0.0170	207.17	K_2	0.780	227.83
S_1	0.290	25.06	M_8	0.0030	237.53
M_1	0.0140	123.43	$(MS)_4$	0.0120	137.85
J_1	0.0170	121.85			

example, to define the tide at Anchorage, AK, over 100 constituents are necessary.

Two categories of tidal constituents are computed—those that represent the elevation of the water surface and those that specify time. For example, the value for H_n in Equation 1 is the constituent amplitude and is a function of both location and variations arising from changes in the latitude of the moon's node. The moon's nodal effect is reflected by the introduction of the node factor f_n, which modifies each constituent amplitude to correspond to a specific time period. Midyear values are usually specified for a given time series reconstruction because node factors vary slowly in time. Midyear values for each constituent of Table 1 are published for the years of 1850 to 1999 (Schureman 1958).

The second category of arguments specify the timing of the individual constituent high-water mark with respect to both local time and global time. These arguments are based on the fact that phases of the constituents of the observed tide do not coincide with the phases of the corresponding constituents of the equilibrium tide. For example, a high tide does not occur directly beneath the moon. There is a lag between the high-water phase of the argument (that is, location of the moon) and the observed time of high water. This lag is referred to as the epoch of the constituent and is denoted by κ in Equation 1.

The relationship between the constituent argument and the corresponding high tide is shown in the schematic diagram of Figure 2. In this figure, time is increasing to the right, and κ represents the phase lag or time required for the water surface to reach high water (HW) following the crossing of the moon (M). Because the water does not respond exactly according to theory, the value of κ is computed as the sum of the theoretical argument $(V_0 + u)$ and the actual observed phase angle ζ at some time T.

Because constituents can be considered harmonic (can be expressed as a cosine function whose argument increases linearly with time), the value of κ is relatively constant at every location. That is, the value of κ represents the actual lag between the tidal potential and the following high tide as a function of observational data adjusted to reflect time to equilibrium theory. Therefore, the value of κ can be computed from the value of ζ derived from prototype data measured at any time and the corresponding adjustment according to $(V_0 + u)$ for that location at that time. Values of the equilibrium argument for the constituents of Table 1 relative to the passing of the tidal potential at the Greenwich meridian are published for each calendar year from 1850 through 2000 (Schureman 1958).

Phases of a given tidal constituent in different parts of the world are not directly comparable with respect to the local epoch κ because κ is a function of the longitude of the specific location. However, an adjusted

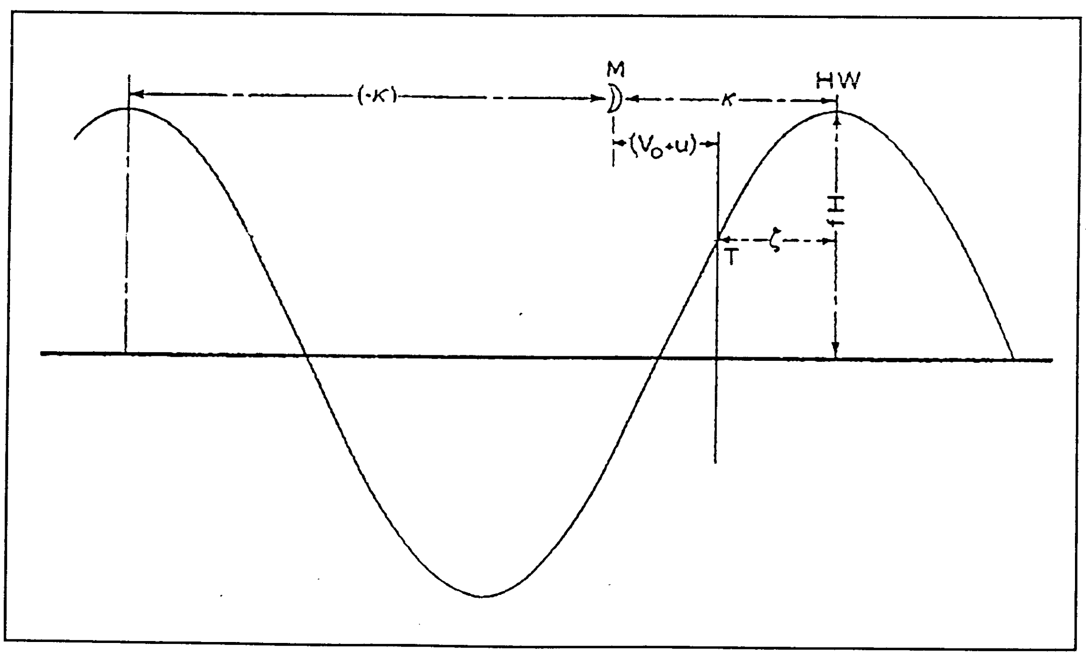

Figure 2. Tidal phase relationships

epoch can be computed which is independent of both longitude and the time meridian. This epoch has been designated as the Greenwich epoch, G, and is related to the local epoch κ of Equation 1 according to the following:

$$\textit{Greenwich epoch } (G) = \kappa + pL \tag{2}$$

where p is the coefficient indicating the number of cycles per day (1 for diurnal and 2 for semidiurnal) and L is the longitude of the station. The epochs of the DRP database are Greenwich epochs because the time series generated by the ADCIRC model are computed with boundary conditions referenced to the Greenwich epoch (Schwiderski 1980).

The following section demonstrates use of the DRP database to manually generate an equilibrium surface elevation time series of data at any location of the computational domain shown in Figure 1. This application provides the user a rapid capability of estimating tidal elevation data at any location without use of computer support.

Tidal Surface Elevation Time Series—Manual Reconstruction

One application of the database is to provide a means of specifying a realistic tidal elevation at some specific location but not to provide a tide prediction that is accurate in both magnitude and time of high and low tide. Because the precise time of arrival of the tide is not important in a long-term disposal site stability application, only the Greenwich epochs are used. Therefore, the following example demonstrates the generation of an equilibrium tide in which the nodal factor f is 1.0, the equilibrium argument $(V_0 + u)$ is 0.0, and the local epoch κ is replaced by the Greenwich epoch G. However, if a tidal hindcast or prediction is desired and computer resources are not available, guidelines for computing the node factor and equilibrium arguments are given in Schureman (1958), and the values of κ can be computed from the values of G according to Equation 2. If computer resources are available, guidelines for generating time series of tidal elevations and currents are provided in the next section.

To reconstruct the tide for any location, the values of the amplitudes and Greenwich epochs for a particular location must be extracted from the database. For the manual reconstruction approach, detailed co-tidal charts are provided in Westerink, Luettich, and Scheffner (1993). An example of the M_2 charts for the east coast is shown in Figure 3 for amplitude and in Figure 4 for Greenwich epoch. The steps described in the following paragraphs are performed to generate, or resynthesize, a tidal signal.

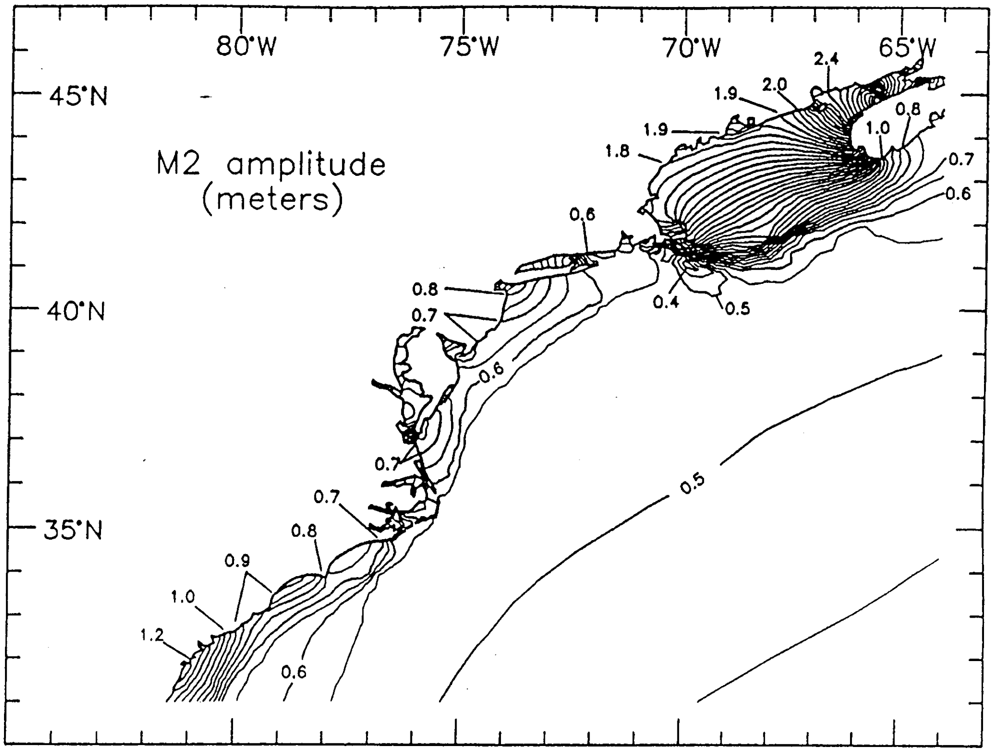

Figure 3. Computed contours for the M2 amplitude (in meters)

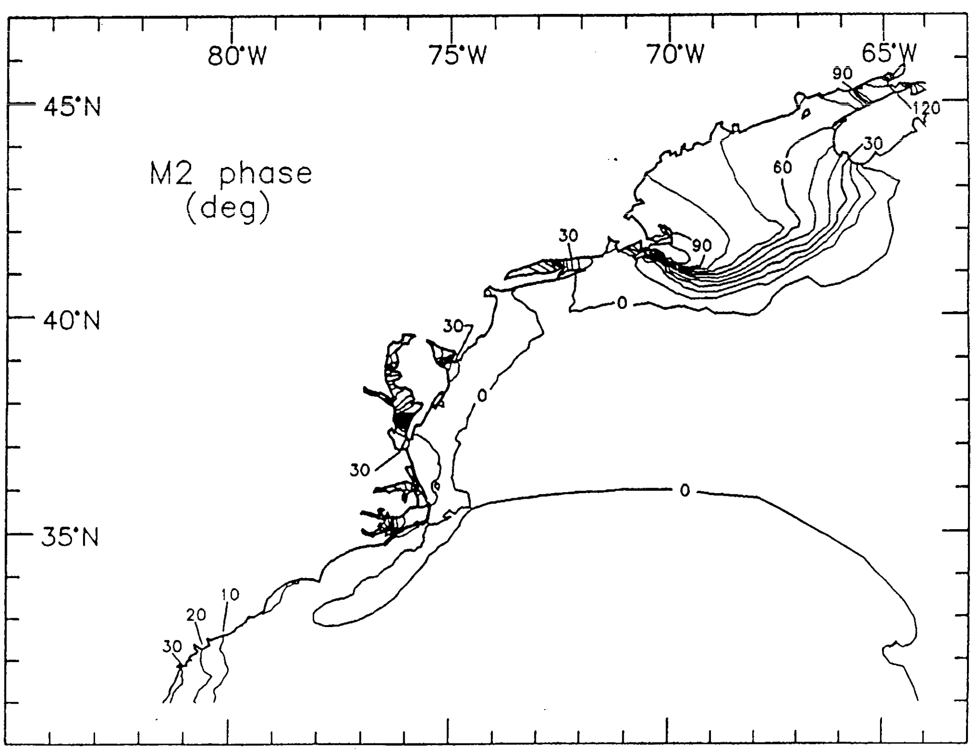

Figure 4. Computed contours for the M2 Greenwich epoch (in degrees)

Step 1

Interpolate amplitudes and phases for the eight astronomical constituents from the co-tidal charts. For the Duck, NC, example presented above, the amplitudes and Greenwich epochs shown in Table 2 were extracted from Westerink, Luettich, and Scheffner (1993). The constituent speeds shown in the table are readily available from sources such as Schureman (1958) or Dronkers (1964).

Table 2. Harmonic Arguments for Duck, North Carolina

Constituent	Amplitude H, m	Epoch *G*, deg	Speed ω, deg/hr
K_1	0.092	179	15.0410686
O_1	0.066	183	13.9430356
P_1	0.038	175	14.9589314
Q_1	0.012	179	13.3986609
N_2	0.14	354	28.4397296
M_2	0.63	14	28.9841042
S_2	0.11	57	30.0000000
K_2	0.037	48	30.0821372

Step 2

Compute the tide signal according to Equation 1. The resynthesized tidal elevation signal for the data between days 50 and 60 is shown in Figure 5.

Tidal Surface Elevation and Current Time Series—Computer Generation

Tidal time series based on the surface elevation and current (u and v components) amplitude and the Greenwich epoch database are available for mainframe or PC applications. The computer program that accesses these data has the capability of computing equilibrium arguments (Equation 1) so that hindcast or forecast time series can be generated. Therefore, either equilibrium time series or time-referenced time series can be generated. Input to the program includes equilibrium or hindcast/forecast option, length of desired time series and time increment between data points, location latitude and longitude if a hindcast/prediction is desired, and starting time, in hour, day, month, and year.

The program-generated prompts for a 30-day tidal hindcast at Duck, NC (latitude 36.1833, longitude 75.7333), beginning at 0000 hr on 1 July 1993, are shown in Figure 6. The generated time series is shown in Figure 7. Node factors, equilibrium argument amplitudes, and epochs (both

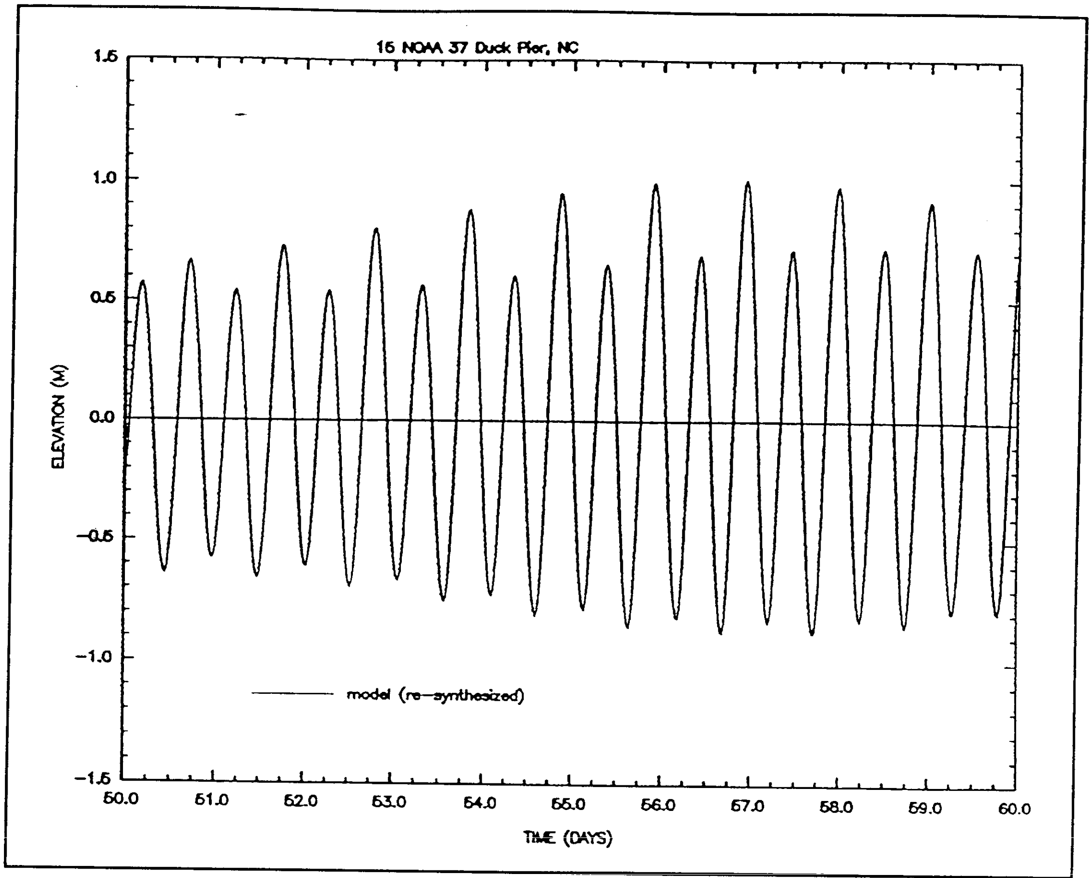

Figure 5. Manually computed equilibrium tidal elevation time series for Duck, NC

```
h2crpns0:larry$ tidepred.x
  ENTER 0 FOR EQUILIBRIUM TIDE G(V0 + U)=0,F=1
  OR 1 FOR TIDAL HINDCAST/PREDICTION
1
 ENTER DESIRED DELT(HRS), LENGTH OF TIME SERIES(DAYS)
1.,30

  ENTER THE LONGITUDE AND LATITUDE FOR TIDAL INFORMATION OUTPUT
  LONGITUDE (DEG E OF GREENWICH IS +,  DEG W OF GREENWICH IS -):
-75.7333
  LATITUDE (DEG N OF EQUATOR IS +,  DEG S OF EQUATOR IS -):
36.1833


 OUTPUT WILL BE GENERATED FOR THE POSITION:
      -75.73330 E LONGITUDE ,   36.18330 N LATITUDE


 ENTER STARTING TIME - BHR,IDAY,IMO,IYR
0.,1,7,1993
  30 DAY TIDE PREDICTION STARTING:  HR- 0.00,  DAY-  1,  MONTH-  7  YEAR- 1993

STOP  (called by $MAIN )
CP: 4.288s,  Wallclock: 149.492s,  0.4% of 8-CPU Machine
HWM mem: 183447, HWM stack: 2048, Stack overflows: 0
```

Figure 6. Computer prompts for Duck, NC, tide hindcast for July 1993

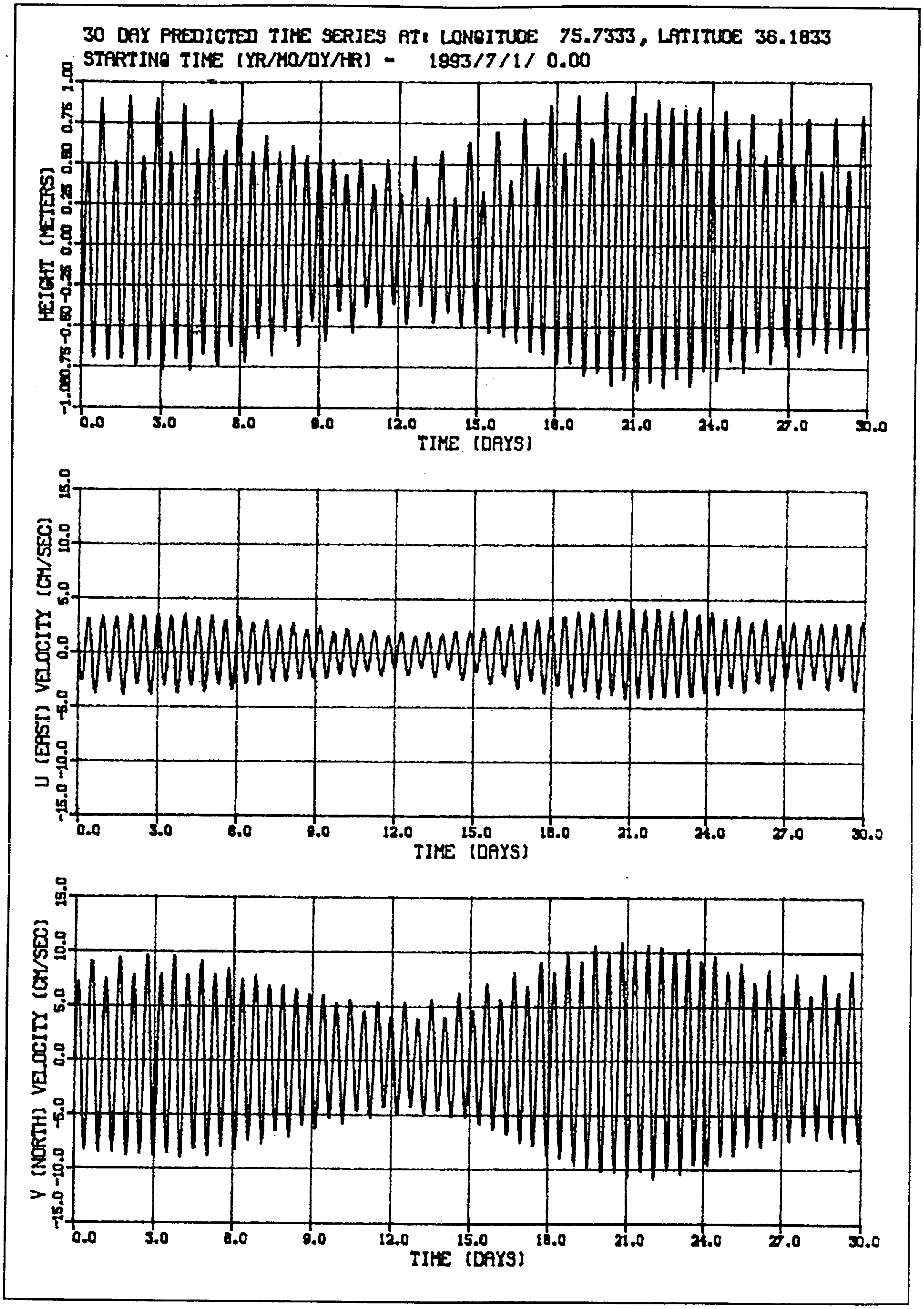

Figure 7. Computer-generated tidal elevation and current time series for Duck, NC, July 1993

κ and G), and time series used to generate Figure 7 are contained in a file named **tide.plot**, shown in part in Figure 8.

Conclusions

A database of tidal elevation and current astronomical arguments has been completed for the east coast of the United States, Gulf of Mexico, and Caribbean Sea. Co-tidal charts of surface elevation amplitudes and Greenwich epochs for the east coast and Gulf of Mexico are provided in Westerink, Luettich, and Scheffner (1993). That publication provides the capability of generating tidal elevation time series for any location included in the charts and does not require the use of a computer. The full database of surface elevation and current amplitudes and epochs, as well

```
h2crpns0:larry$ pg tide.plot
  1.0 HOUR TIME INCREMENT     1  75.73330  36.18330

   30 DAY TIDE PREDICTION STARTING:  HR- 0.00,  DAY-  1,  MONTH-  7  YEAR- 1993

 CONST NODE      EQ ARG   ELEVATION           EAST VELOCITY          NORTH VELOCITY
 NAME FACTOR    (DEG)  AMP(M) G(DEG)K(DEG) AMP(M) G(DEG)K(DEG) AMP(M) G(DEG)K(DEG)

 K1  0.97400 197.79 0.0923 179.4 103.6  0.0025 327.5 251.8  0.0048 141.2  65.5
 O1  0.95771 242.59 0.0659 183.6 107.8  0.0017 338.1 262.4  0.0033 154.0  78.2
 P1  1.00000 170.94 0.0381 175.5  99.8  0.0010 312.2 236.5  0.0020 125.4  49.6
 Q1  0.95771 183.51 0.0121 179.3 103.5  0.0002 340.6 264.8  0.0004 154.8  79.0
 N2  1.01291  25.46 0.1397 354.0 202.6  0.0071 114.7 323.3  0.0179 282.7 131.3
 M2  1.01291  84.54 0.6293  13.7 222.2  0.0283 132.8 341.3  0.0743 299.7 148.3
 S2  1.00000 360.00 0.1119  57.2 265.7  0.0063 173.3  21.9  0.0161 342.6 191.1
 K2  0.91871 215.28 0.0366  48.4 256.9  0.0020 163.4  11.9  0.0052 332.8 181.3

 TIME (HRS) AMPLITUDE (M) EAST VELOCITY (M/SEC) NORTH VELOCITY (M/SEC)

      0.000  -0.66711             0.00847             -0.00119
      1.000  -0.50549            -0.00623              0.03571
      2.000  -0.23206            -0.01866              0.06231
      3.000   0.07807            -0.02563              0.07179
      4.000   0.34191            -0.02533              0.06172
      5.000   0.48947            -0.01782              0.03456
      6.000   0.48150            -0.00500             -0.00282
      7.000   0.31946             0.00984             -0.04092
      8.000   0.04525             0.02287             -0.06993
      9.000  -0.26915             0.03063             -0.08220
     10.000  -0.53979             0.03094             -0.07419
     11.000  -0.69186             0.02346             -0.04733
     12.000  -0.67884             0.00978             -0.00780
     13.000  -0.49465            -0.00694              0.03500
     14.000  -0.17586            -0.02272              0.07078
     15.000   0.20672            -0.03378              0.09081
     16.000   0.56500            -0.03742              0.09017
     17.000   0.81500            -0.03272              0.06891
     18.000   0.89741            -0.02077              0.03211
     19.000   0.79224            -0.00440             -0.01138
     20.000   0.52379             0.01248             -0.05111
     21.000   0.15483             0.02588             -0.07759
     22.000  -0.22864             0.03268             -0.08464
     23.000  -0.53840             0.03140             -0.07090
     24.000  -0.70549             0.02258             -0.04018
```

Figure 8. Sample listing of tide data file

as the capability of computing equilibrium arguments for tidal hindcasts or predictions, is available to users with mainframe or PC capabilities.

The intended purpose of the database is to generate time-independent realistic tidal data for a specific location for use as boundary conditions for the long-term fate model LTFATE. However, tidal elevation and current hindcast or prediction capabilities are available as an alternative to published tide tables that give only high and low tide predictions. The DRP database therefore represents an improvement to the tide tables because it provides a capability of generating a continuous tidal signal for any time period at any location in the computational domain shown in Figure 1.

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