How humans deal with uncertainty in decisions

Lev Ginzburg and Scott Ferson, Applied Biomathematics

Rationality

• Maximize expected utility (EU)
  – Pascal 1670; von Neumann & Morgenstern 1944
  – If making many such decisions, EU performs best

• Your probabilities must make sense
  – *Coherent* if bets don’t expose you to sure loss
So why not use EU?

- Not how people act
- Needs a lot of information to use
- Probabilities are often inconsistent
- Unsuitable for important unique decisions
- Inappropriate if gambler’s ruin is possible

Risk aversion

- **Gamble:**
  - EITHER get $50
  - OR get $100 if a randomly drawn ball is red from urn with half red and half blue balls
- Which prize do you want?

$50

EU is the same, but most people take the sure $50
Ambiguity aversion

- Balls can be either red or blue
- Two urns, both with 36 balls
- Get $100 if a randomly drawn ball is red
- Which urn do you wanna draw from?
Montauk workshop

- Several modules in human brain
  - Strong clinical and neuroimaging evidence
  - Examples: cheater detection, language, etc.
  - Each is tuned to a specific kind of problem
  - Activated by sensory input in a particular format
  - Modules sometimes disagree with each other

- Irrationality is a hallmark of human decisions
  - Suicide bombers, philanthropy, lotteries, altruism, war, etc.
  - Other primates sometimes more rational

- Emotions crucial to decision making

Import for risk assessment

- Risk analyses woefully incomplete
  - Omit important issues and thus understate risks
  - Neglect or misunderstand incertitude

- Presentations use very misleading formatting
  - Percentages, relative frequencies, conditionals, etc.

- Both problems can be fixed
  - By changing analysts’ behavior, not the public’s
Imprecision about probabilities

- Bayesian “rational agents” are compelled to either sell or buy any offered bet
  - But it’s not irrational to decline to bet

- Imprecise probabilities don’t require all bets
  - Interval probability for A is the range between the largest buying price and the smallest selling price you accept for the bet that A happens

Probability bounds analysis

Convenient and comprehensive software available for arithmetic, logic, comparisons and analysis
Interval dominance

- Dominant
- Inadmissible
- Overlap

Red is preferred to blue
Red is incomparable to blue

Bridges Knight’s decisions

1929

Under risk
- Probabilities known
- Maximize expected utility
- EU

Under uncertainty
- Probabilities unknown
- Several possible strategies
- Maximin, Wald 1939

Under imprecision
- Probabilities partially known

Ginzburg and Ferson, Applied Biomathematics
Different decision criteria

• Some criteria find a single decision
  – Γ-maximin picks the decision with the highest low

• Some criteria can yield sets of decisions
  – E-admissibility picks all decisions that may be best
  – The more precise the input, the tighter the outputs

• Different criteria are useful in different settings

Traditional Bayesian answer

• Allows only one decision (up to indiﬀerence)
  – No matter how much uncertainty is present

• Diﬀerent analysts would get diﬀerent answers
  – Depends on which prior we use for \( p \)

• Doesn’t express doubt about the final decision
Take-home messages

- Improper to say you know more than you do
- Bayesian decision making always yields one answer, even if it is not really justified
- Prudent decisions under imprecision
  - More consistent with human psychology
  - Tells you when to reserve judgment

Applied Biomathematics research

- NSF      Risk perception as neuroscience and anthropology
- Pfizer   Perspective visualization of data through uncertainty
- Sandia   Accounting for epistemic uncertainty
- DHS      Spatial risk maps from imperfect incidence data
- NASA     Aleatory and epistemic uncertainty in spacecraft design
- BRGM     Bayesian methods in risk assessment
- NIH      Detecting clustering of rare events in sparse data
- USDA     Forest pest risk in dynamic landscapes
Ellsberg Paradox

- Balls can be red, black or yellow (probs are $R, B, Y$)
- A well-mixed urn has 30 red balls and 60 other balls
- Don’t know how many are black, how many are yellow

<table>
<thead>
<tr>
<th>Gamble A</th>
<th>$R &gt; B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $100$ if draw red</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Gamble B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $100$ if draw black</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gamble C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $100$ if red or yellow</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gamble D</th>
<th>$R + Y &lt; B + Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get $100$ if black or yellow</td>
<td></td>
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</tbody>
</table>
Persistent paradox

- Most people prefer A to B (so are saying \( R > B \)) but also prefer D to C (saying \( R < B \))
- Doesn’t depend on your utility function
- Payoff size is irrelevant
- Not related to risk aversion
- Evidence for ambiguity aversion
  - Can’t be accounted for by EU
  - Not resolved in Prospect Theory either

Dutch book example

<table>
<thead>
<tr>
<th>Horse</th>
<th>Offered odds</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danger Spree</td>
<td>Evens</td>
<td>0.5</td>
</tr>
<tr>
<td>Windtower</td>
<td>3 to 1 against</td>
<td>0.25</td>
</tr>
<tr>
<td>Shoeless Bob</td>
<td>4 to 1 against</td>
<td>0.2</td>
</tr>
</tbody>
</table>

A gambler could lock in a profit of 10, by betting 100, 50 and 40 on the three horses respectively

Comparing IP to Bayesian approach

- Axioms identical except IP doesn’t use completeness

- Bayesian rationality implies not only avoidance of sure loss & coherence, but also the idea that an agent must agree to buy or sell any bet at one price

- “Uncertainty of probability” is meaningful, and it’s operationalized as the difference between the max buying price and min selling price

- If you know all the probabilities (and utilities) perfectly, then IP reduces to Bayes

Why Bayes fares poorly

- Bayesian approaches don’t distinguish ignorance from equiprobability

- Neuroimaging and clinical psychology shows humans strongly distinguish uncertainty from risk
  - Most humans regularly and strongly deviate from Bayes
  - Hsu (2005) reported that people who have brain lesions associated with the site believed to handle uncertainty behave according to the Bayesian normative rules

- Bayesians are too sure of themselves (e.g., Clippy)
Multi-criteria decision analysis

- Used when there are multiple, competing goals
  - E.g., USFS’ multiple use (biodiversity, aesthetics, habitat, timber, recreation, …)
  - No universal solution; can only rank in one dimension
- *Group* decision based on *subjective* assessments
- Organizational help with conflicting evaluations
  - Identifying the conflicts
  - Deriving schemes for a transparent compromise
- Several approaches
  - Analytic Hierarchy Process (AHP); Evidential Reasoning; Weight of Evidence (WoE); *every approach has flaws*
Analytic Hierarchy Process

- Identify possible actions
  - buy house in Setauket / buy in Port Jeff / rent in Old Field
- Identify and rank significant attributes
  - location > price > school > near bus
- For each attribute, and every pair of actions, specify preference
- Evaluate consistency (transitivity) of the matrix of preferences by eigenanalysis
- Calculate a score for each alternative and rank

- Subject to rank reversals (e.g., without Perot, Bush beat Clinton)